THE UNILATERAL DYNAMIC CHARACTERISTICS OF THREE ELECTRODE THERMIONIC AMPLIFIERS

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Introduction

So many papers have been published in recent years describing the general fundamental principles and operations of vacuum thermionic amplifiers that this phase of the subject will be passed over here. For the purpose of this paper it is sufficient to know that a thermionic amplifier consists of a filament, which can be heated by an external battery and become a source of electrons, an anode, usually called the plate which may be either cylindrical or consist of two parallel plates, an auxiliary electrode, usually called a grid, which is placed between the filament and the anode, and which owes its name to the fact that it consists of a mesh of fine wires with rectangular apertures. Connections from the grid, plate and filament are led out through the glass bulb, in which these parts are mounted, the bulb being evacuated to the highest possible vacuum obtainable. When a potential is applied between plate and filament and the latter heated to a sufficiently high temperature an electronic current will flow from filament to plate, provided the plate has a positive potential with respect to the filament. The current from filament to plate may be controlled by varying the difference of potential between grid and filament. If the potential of the grid is sufficiently high, it may draw all of the electrons to itself from the plate. On the other hand if it is low then the potential will tend to prevent electrons getting through it. In both cases therefore the current to the plate will be diminished.

Curves showing the relation of this electronic current to the grid and plate voltages are usually called the characteristic curves of the amplifier. A study of the static characteristics, that is when steady potentials and currents are considered, has been made in recent years by Langmuir,[1] Bethenod,[2] Vallauri,[3] Vander Bijl,[4] Latour,[5] and others. The main object of their researches was to obtain a mathematical expression showing how the current from the filament to the plate depended on the grid and plate voltages.

When alternating grid and plate potentials are used, the corresponding curve is known as the dynamic characteristic. This curve is a function not only of the grid and plate voltages but also of the internal and external current circuits. Van der Bijl,[6] using the same law for instantaneous currents and potentials as in the static case, has shown how a pure sine wave of e.m.f. impressed between grid and filament affects the output plate current, in cases where there is not and where there is an external resistance in the plate circuit. Haseltine[7] has shown how to obtain the dynamic characteristic graphically in the case where the tube is used as a source of persisent oscillations. He terms this the derived characteristic, as it is derived from a knowledge of the alternating e.m.f's impressed on grid and plate due to the coupling with the oscillatory circuit. Lewis M. H~~ø~~ll,[8] making use of the "derived" characteristic has measured the amplitudes of the harmonics in the oscillating circuit of a vacuum tube generator. Blondell[9] has obtained a general mathematical relation for the current in an oscillating circuit when a pure sine wave e.m.f. is impressed between grid and filament.

Van der Bijl[10] has shown theoretically that in the case where a resistance

1. P.I.R.E. 3, 261-93 Sept. 1915 and Phys. Rev. 2, P. 457, 1913
2. La Sum. El. 35, 25-31 Oct. 14, 1916
3. L'Elettrotecnica Vol. 4 Nos 3,4,18 and 19, 1917
4. Phys. Rev., 11, p 172-198, 1918
5. La Sum. El. Dec. 30, 1916
6. Loc. Cit.
7. P. I. R. E. April 1918
8. Bureau of Standards. Scientific Papers. No. 355. Dec. 1, 1919
9. Comptes Rendues t 169, p 676 et 820 et p 943
10. Loc. Cit

is inserted in the plate circuit, the resulting dynamic characteristic is no longer of the parabolic shape of the static characteristic when the resistance was absent, but approaches more nearly to a straight line as the resistance is increased. The fact that the characteristic is straightened out, tends to decrease the amplitudes of the higher harmonics. If the characteristic curve were a straight line the fundamental frequency would alone persist.

With the object of testing the conclusions of Van der Bijl the author has undertaken to measure the actual amplitudes of the fundamental and harmonic constituents of the output current wave for the case when a pure sine wave e.m.f. is impressed on the grid, and a pure resistance is inserted in the plate circuit. A mathematical expression has also been worked out for the cases where an inductance and a capacity are placed in the plate circuit, and experimental values for the case of the inductance have been obtained.

Fundamental Theory

Properties of the Static Characteristic.

For completeness, it may be well to give a few of the formulae applyèng to the static charasteristic. Most of these are, of course, well known, but a few present certain original features of demonstration.

Writing $I_b = F(E_b, E_c)$ (1)

$I_c = f(E_b, E_c)$

where E_b = plate potential

see figure I

E_c = grid potential

I_b = plate current

I_c = grid current

H. W. Nichols[1] has shown that the following relations hold

$$dE_b = R\, dI_b + \frac{dE_b}{dI_c}\, dI_c \qquad (2)$$

$$dE_c = \frac{dE_c}{dI_b}\, dI_b + r\, dI_c \qquad (3)$$

where $R = \dfrac{dE_b}{dI_b}$ and $r = \dfrac{dE_c}{dI_c}$

1. Phys. Rev. 13 p 406 - 407 1919

It has been assumed in this derivation that I_b and I_c are within certain limits finite, single-valued continuous functions of E_b and E_c. E_b must always be taken positive and E_c may be either positive or negative within certain limits.

The coefficient $\frac{\partial E_c}{\partial I_b}$ may be made very small by removing all external coupling between plate and grid circuits, and provided the capacity between the electrodes is sufficiently small and the frequency of operation is not over say 10^6 cycles per sec., it may be neglected entirely. However, in the case of the plate potential, the term $\frac{\partial E_b}{\partial I_c} \, dI_c$ amounts to a fictitious e.m.f. introduced into the plate circuit by the change I_c in the grid current. If a change in grid current is thus regarded as being capable of producing an apparent change in the plate potential by virtue of the design of the device, but if the latter be denied the power to produce a change in plate current which will affect the grid potential, the tube is said to be used as a means of unilateral amplification.

It is known from the manner of operation of a tube that the plate current may be written as a function of a single variable $E_b + \mu E_c$.

Then $I_b = \Phi\,(E_b + \mu E_c)$, where μ is a constant dependent on the tube.

Therefore $\frac{\partial I_b}{\partial E_b} = \Phi'(E_b + \mu E_c)$

and $\frac{\partial I_b}{\partial E_c} = \mu \Phi'(E_b + \mu E_c)$

$$\text{Therefore} \quad \frac{\frac{\partial I_b}{\partial E_b}}{\frac{\partial I_b}{\partial E_c}} = \frac{1}{\mu}$$

From equation (1)

$$dI_b = \frac{\partial I_b}{\partial E_b} dE_b + \frac{\partial I_b}{\partial E_c} dE_c .$$

Set $dI_b = 0$

$$\text{Therefore} \quad \frac{\frac{\partial I_b}{\partial E_b}}{\frac{\partial I_b}{\partial E_c}} = -\left(\frac{dE_c}{dE_b}\right)_{I_b}$$

$$\text{Therefore} \quad -\left(\frac{dE_c}{dE_b}\right)_{I_b} = \frac{1}{\mu} .$$

Hence $E_b - \mu E_c + e = 0$ at constant I_b.

This result has been deduced by Van der Bijl[1] but in a less general procedure. Consequently, μ may be defined as slope of the E_b, E_c curve when I_b is constant. In physical terms μ is the constant when multiplied by the grid voltage gives the plate voltage to which that on the grid is equivalent. This relation is easily verified experimentally, indicating that the above functional form is a necessary one for the plate current.

Langmuir[2] has shown from theoretical considerations that this function could be written in the form

$$I_b = A\,(E_b + \mu E_c)^{3/2}$$

and that this relation is independent of the geometrical shapes of the tube-elements.

Van der Bijl, however, using parallel grids and plates and an oxide coated platinum filament as a source of electrons has shown that the empirical formula

$$I_b = A\,(E_b + \mu E_c + \varepsilon)^2$$

where A and ε are constants for any particular tube holds very approximately over a certain range of grid and plate potentials. This parabolic relation will be made use of throughout this paper. In this equation E_b must be positive and E_c may be negative or positive within certain limits. If the grid is raised to a very high positive potential electrons will tend to reach it rather than pass through to the plate. In general the maximum positive voltage to which a grid may be raised increases with increasing plate voltage. The grid voltage can only be made negative to an extent given by $|E_b + \varepsilon| - \mu |E_c| \geqq 0$.

Recent papers by H. W. Nichols,[3] J. M. Miller,[4] G. Breit,[5] and Ballantine[6] have shown that the inter-electrode capacities must be taken into consideration when determining such quantities as input impedance and detecting efficiencies.

1. loc. cit.
2. loc. cit.
3. loc. cit.
4. Bureau of Standards Scientific Papers 351, No. 21, 1919
5. Phys. Rev. 16 274-281 1920
6. Phys. Rev. 15 409-420 1920

These capacities will not, however, alter in any way the functional form of the current voltage relation, but will merely diminish or increase the individual terms arising in the relation.

Properties of the Dynamic Characteristic.

The meaning of the term "dynamic characteristic" may be obtained from the following relation

$$I_b' = F(E_b' + \quad E_c') \tag{7}$$

where the primed letters refer to the instantaneous values of currents and potentials. Now if the assumption is made that this function is the same as that already used for steady currents and potentials, then the equation connecting instantaneous currents and potentials is the same as that for the steady state. This does not necessarily mean that when the alternating plate current is plotted against the alternating grid voltage that the same curve will be found as in the steady state. For, now alternating current phenomena are being dealt with, and inductances and capacities will not affect alternating and direct currents in the same manner. In other words the assumption is that as far as the tube is concerned, neglecting inter-electrode capacities, and ultra-radio frequencies where other complications set in, it does not know the difference between the static and dynamic cases, the only thing that shows the difference being the nature of the external circuits through which the pulsating current passes. If the emission of the electrons from the filament were in some way controlled by the rapidly changing grid potential, then the above assumption would no longer be correct. In that case the time would enter into the equation for the current. Experimental evidence, however, does not seem to point to any effect of time in the operation of an amplifier.

<u>Case of no external resistance in plate circuit</u>

$$I_b = A(E_b + \mu E_c + \mathcal{E} + \mu e \sin pt) \tag{8}$$

In this case the plate potential has the steady value E_b and the grid potential has the value $E_c + e \sin pt$. The equivalent plate voltage is therefore $\mu(E_c + e \sin pt)$.

Before proceeding further it might be well to remark here that in order that (8) may actually represent the true conditions the value of "e" must lie within certain limits, namely

$$e \leqq \left| E_c \right| + \left| g \right| \qquad (9)$$
$$e \leqq \left| \frac{E_b + \varepsilon}{\mu} \right| - \left| E_c \right|$$

where g is the maximum positive voltage the grid can have before it begins to attract many electrons. Also if "e sin pt" attains such a large negative value in the cycle that the expression above is negative, the resulting current wave will be flattened out at that part of the characteristic curve. Equation (8) might be written generally as

$$I_b = f\,(\mu e \sin pt)$$

Expanding as a Maclaurin Series

$$I_b = f(0) + \mu e \sin pt\, f'(0) + \frac{\mu^2 e^2 \sin^2 pt}{2!} f''(0)$$

$$f'(0) = \frac{\partial I_b}{\partial E_b} = 2A\,(E_b + \mu E_c + \varepsilon) = \frac{1}{R}$$

$$f''(0) = \frac{\partial^2 I_b}{\partial^2 E_b} = 2A$$

Therefore $I_b = A(E_b + \mu E_c + \varepsilon)^2 + \frac{\mu e \sin pt}{R} - \frac{A\mu^2 e^2}{2} \cos 2pt + \frac{A\mu^2 e^2}{2}$

Thus in the simple case illustrated above where the plate potential is kept constant throughout the operation, a pure sine wave on the grid gives rise to a current of the same frequency (called the fundamental) in the plate circuit, a first harmonic and a rectified current component. In this case case the actual dynamic and static characteristic curves will coincide.

<u>Case of a pure resistance</u>

Next we shall consider the case where the potentials on the grid and plate vary simultaneously. Let a resistance R be connected between the plate and the plate battery. Then

$$I_b = A(E - R\,I_b + \mu(E_c + e \sin pt) + \varepsilon)^2$$

This ~~case~~ can be expanded as an infinite series.

$$I_b = f(0) + f'(0)\,\mu e \sin pt + \frac{f''(0)}{2!}(\mu e \sin pt)^2 + \text{------} \frac{f^n(0)}{n!}(\mu e \sin pt)^n \quad (11)$$

Van der Bijl has shown that since the parabolic relation connecting plate current and grid and plate potentials is only an empirical approximation, it is not to be expected that the higher derivates in the series will accurately represent the actual experimental values. However, the derivatives up to probably the third or fourth ought to be a close approximation and the higher derivatives ought to indicate, at least in a qualitative way, how the higher harmonics depend on the various tube constants and on the properties of the external circuits.

Referring back to equation (11), the coefficient of the series are as follows:

$$f(0) = \frac{1}{2AR^2}\left\{\frac{1}{2}(B+1) - B^{1/2}\right\}$$

where $B = 1 + \dfrac{2R}{R_o}$

R_o being defined as $\dfrac{1}{2A(E + \mu E_c + \varepsilon)}$

$$f'(0) = \frac{1}{R}\left\{1 - B^{-1/2}\right\}$$

$$f''(0) = A\left\{B^{-3/2}\right\}$$

$$f'''(0) = -2A^2R\left\{B\right\}^{-5/2}$$

The general functional term is given by

$$f^n(0) = \frac{(-1)^n\, 2^{n-1}\, n(n-2)(n-4)\text{-----}3\ R^{n-2}A^{n-1}}{n!\,B^{\frac{2n+1}{2}}}$$

The coefficient of sin(pt) is therefore the value of this expression multiplied by $(\mu e)^n$. If this coefficient is denoted by

$$\text{Then } \frac{|\alpha_{n+1}|}{|\alpha_n|} = \frac{2(n+2)RA\mu e}{(n+1)B}$$

$$\lim_{n=\infty} \frac{|\alpha_{n+1}|}{|\alpha_n|} = \frac{2RA\mu e}{B}$$

In order that the series (11) may be absolutely convergent

$$\frac{2RA\mu e}{B} < 1$$

$$\text{Therefore } e < \frac{B}{2RA\mu} \quad \text{i.e.} < \frac{1}{2RA\mu} + \frac{1}{R_oA\mu}$$

$$\text{Therefore } e < \frac{1}{2A\mu}\left\{\frac{R_o + 2R}{2RR_o}\right\} \quad (12)$$

Thus for a given A, μ and R, the smaller the value of R_o, the greater e may be.

Using the values of A, μ, R and R_o given later, e may have values reaching up to 150 volts. However, it will be seen later that in practice e cannot have a value larger than about 15 volts if equation (10) is to accurately represent conditions. The physical limitations which the tube imposes on the characteristic equation make it impossible to use grid voltages more than one-tenth of the limiting value as given by (12). It is very evident that for small values of e, that the series (11) converges rapidly and in consequence only a few terms may be evaluated in order to find a close approximation to the actual current flowing under a certain condition of the amplifier.

Now $f'(0)$ stands for the reciprocal of the total output resistance when there is an external resistance in the plate circuit.

$$\text{Therefore } \frac{1}{R_o'} = \frac{1}{R}\left\{1 - B^{-1/2}\right\}$$

The total resistance of the complete plate circuit is thus a rather complicated function of the external resistance and the internal output resistance of the tube when there was no resistance in the plate circuit.

Since the series (11) is a power series in sin(pt) it is necessary to convert the various powers of sin(pt) into first power expressions of functions of multiples of pt, and expressions corresponding to the rectified currents. Since the series converges rapidly for values of input voltage within the limits (9), all powers of sin(pt) beyond the fourth will be omitted.

$$\begin{aligned} I = {} & \frac{1}{2AR^2}\left\{\tfrac{1}{2}(B+1) - B^{\frac{1}{2}}\right\} \\ & + \left[\left\{\frac{\mu e}{R}\left(1 - B^{-1/2}\right)\right\} - \frac{3}{8} A^2 R \mu^3 e^3 B^{-5/2}\right] \sin(pt) \\ & - \frac{1}{2} A \mu^2 e^2 B^{-3/2} + \frac{5}{8} A^3 R^2 \mu^4 e^4 B^{-7/2}\Big] \cos(2\,pt + \pi) \\ & + \frac{1}{8}\left[A^2 R \mu^3 e^3 B^{-5/2}\right] \sin 3\,pt \\ & + \frac{5}{8}\left[A^3 R^2 \mu^4 e^4 B^{-7/2}\right] \cos 4\,pt \\ & + \text{- - - - - - - - - - - - - - - -} \end{aligned}$$

Actual computation of the coefficients in this series show that for values of "e" within the limits specified above, the series converges very rapidly.

The following computations will give an idea of the order of magnitude of these coefficients for a certain condition.

Let $R = 2700$ ohms.

" $E = 260$ Volts

" $E_c = -7.5$ Volts

" $R_o = 3750$ ohms.[1]

$$\left(\frac{2R}{R_o} + 1\right)^{-1/2} = .63$$

Let $e = 1$ volt

Therefore $\mu e = 6.7$ volts, since $\mu = 6.7$ for the tube.

Coefficinet of sin pt $= (9.18 \times 10^{-4} - 1.91 \times 10^{-8})$ ampere

Coefficient of cos $(2\,pt + \pi) = (3.08 \times 10^{-6} - .20 \times 10^{-10})$ ampere

Coefficient of sin 3 pt $= .635 \times 10^{-8}$ ampere

Coefficient of sin 4 pt $= .51 \times 10^{-10}$ ampere

For values of "e" below 10 volts, the coefficient of sin pt is practically a linear function (e); beyond 10 volts, however, the term involving (e^3) becomes appreciable, and consequently the relation is no longer linear. Similarly the coefficients of cos$(2\,pt + \pi)$ vary as the square of e for values of e up to about 15 volts. For higher values of e, the term involving the fourth power of "e" tends to reduce the amplitudes from the square law value.

Since we have taken no higher powers than $\sin^4 pt$, the coefficients of the third and fourth harmonics vary directly as the cube and fourth powers of e. It will be seen later that for values of "e" over 15 volts, using the plate and grid potential values as above, equation (10) no longer holds, and consequently the amplitudes of the harmonics obtained for values of "e" over 15 volts depend on other features of the amplifier. Since the value of E_c is -7.5 the grid will be

1. The value of R_o can be obtained from the value of $\dfrac{1}{\frac{\partial I_b}{\partial E_c}}$ from the equation $I = A(E_b + \mu E_c + \epsilon)$, A and ϵ being constants.

raised to a positive potential of 7.5 volts during this cycle. In figure 6 it will be noticed that at this value of E_c on the 200 volt parameter, the static characteristic begins to lose its parabolic nature and tends to flatten out. From the nature of the static characteristics it may be seen that the higher the plate voltage is raised the greater the values e may have and remain within the proper limits. This amounts to saying that the smaller R_0 is, the greater the input voltage on the grid may be. This conclusion was arrived at in discussion of the convergence of series (11).

The dynamic characteristic for this case may be obtained as follows. The instantaneous values of the various harmonics for values of pt between o and 2 π are plotted, and then these constituent sine waves compounded to give the actual wave shape. If now the resulting periodic current is plotted along the I_b axis and the input voltage plotted on the E_c axis, the resulting curve will be the so-called dynamic characteristic of the tube under the specific conditions. It will be seen that if all terms but the fundamental had been neglected, the characteristic would have been a straight line. Addition, however, of the first harmonic causes the characteristic to have a definite curvature. The smaller the value of the external resistance, the more nearly does the curve approach the parabolic relation holding in the static case, and, of course, in the limiting case when R is zero, the two characteristics coincide.

Condition for maximum output.

In connection with the expression for the internal resistance, it may be pointed out that the usual statement that the maximum power is obtained from a tube when the external resistance in the plate circuit is equal to the internal output resistance of the tube needs clarification. If by maximum power is meant the greatest power obtained from the fundamental frequency, the following is valid.

$$\text{Power} = R\,I^2 = \frac{\mu^2 e^2}{R}\left[1 - B^{-1/2}\right]^2 \qquad (15)$$

Therefore $\frac{dP}{dR} = -\frac{\mu^2 e^2}{R}\left[1 - B^{-1/2}\right]\left[\frac{1}{R}(1 - B^{-1/2}) - \frac{2}{R_o} B^{-3/2}\right]$ (16)

$\frac{dP}{dR} = 0$ for maximum P

Therefore $B^{3/2} = 2B - 1$

or $\frac{R}{R_o} = \frac{1 \pm \sqrt{5}}{4} = .81$ (17)

The condition for a maximum dissipation of fundamental current energy is that the ratio of R to the internal resistance when R was zero is .81. This condition holds in the case where the maximum power is desired with a certain fixed plate battery, and a variable resistance is available.

Case of An Inductance.

When an inductance "l" is placed between the plate and plate battery, the equation for the plate current may be written as follows:

$$I = A\left\{E_b - l\frac{dI}{dt} + \mu(E_c + e\cos pt) + \varepsilon\right\}^2 \quad (19)$$

$$= B^2 - 2BL\frac{dI}{dt} - 2LE\cos pt\frac{dI}{dt} + 2BE\cos pt + L^2\left(\frac{dI}{dt}\right)^2 + E^2\cos^2 pt \quad (20)$$

where $B = A^{1/2}(E_b + E_c + \varepsilon)$

$L = A^{1/2} l$

$E = A^{1/2}\mu e$

A rigorous solution of this differential equation for I is very difficult, but an approximate method of solving it may legitimately be made use of. Experimental evidence shows that I is a rapidly converging Fourier series, and that the frequency of the fundamental is the same as the frequency of the input e.m.f. on the grid.

We can therefore write

$$I = a_{0/2} + \alpha_1 \sin pt + \alpha_2 \sin 2pt + \alpha_3 \sin 3pt + \cdots \alpha_n \sin npt + \cdots$$
$$+ \beta_1 \cos pt + \beta_2 \cos 2pt + \beta_3 \cos 3pt + \cdots \beta_n \cos npt + \cdots \quad (21)$$

But $\alpha_1 \sin pt = \frac{\alpha_1}{2i}(e^{ipt} - e^{-ipt})$ where $i = \sqrt{-1}$

and $\beta_1 \cos pt = \frac{\beta_1}{2}(e^{ipt} + e^{-ipt})$

$\therefore \alpha_1 \sin pt + \beta_1 \cos pt = \frac{e^{ipt}}{2}(\beta_1 - i\alpha_1) + \frac{e^{-ipt}}{2}(\beta_1 + i\alpha_1)$

and similarly for the multiples of pt.

Let $a_1 = \dfrac{\beta_1 - i\alpha_1}{2}$ and so on for $\alpha_2,\ \alpha_3$ --- $\beta_2,\ \beta_3$ ----- (22)

$b_1 = \dfrac{\beta_1 + i\alpha}{2}$

Therefore $I = a_0/2 + a_1 e^{ipt} + a_2 e^{2ipt} + a_3 e^{3ipt} + a_4 e^{4ipt} + \cdots\ b_1 e^{-ipt} + b_2 e^{-2ipt} + \cdots$ (23)

If we substitute this value of I in equation (20) and arrange the result in ascending orders of e^{ipt} and in descending orders of e^{-ipt} we have as follows:

$$
\begin{aligned}
&a_0/2 + a_1 e^{ipt} + a_2 e^{2ipt} + a_3 e^{3ipt} + \cdots + b_1 e^{-ipt} + b_2 e^{-2ipt} + b_3 e^{-3ipt} + \cdots \\
&= B^2 + \frac{E^2}{2} - L^2p^2\{2a_1b_1 + 8a_2b_2 + 18a_3b_3 + 32a_4b_4 \cdots\} \\
&+ e^{ipt}[-2BLip\,a_1 + BE - 2LEip\,a_2 + L^2p^2(4a_2b_1 + 12a_3b_2 + 24a_4b_3 \cdots)] \\
&+ e^{2ipt}[-4BLip\,a_2 + \frac{E^2}{4} - LEip\,(a_1 + 3a_3) - L^2p^2(a_1^2 - 6a_3b_1 - 16a_4b_2 \cdots)] \\
&+ e^{3ipt}[-6BLip\,a_3 - LEip\,(2a_2 + 4a_4) - L^2p^2(4a_1a_2 - 8a_4b_1 - 20a_5b_3 \cdots)] \\
&+ e^{4ipt}[-8BLip\,a_4 - LEip\,(5a_5 + 3a_3) - L^2p^2(6a_1a_3 + 4a_2^2 - 10a_5b_1 \cdots)] \\
&+ e^{5ipt}[\cdots\cdots] \\
&\cdots\cdots \\
&+ e^{-ipt}[2BLip\,b_1 + BE + 2LEip\,b_2 + L^2p^2(4a_1b_2 + 18a_2b_3 + 24a_3b_4 \cdots)] \\
&+ e^{-2ipt}[4BLip\,b_2 + \frac{E^2}{4} + LEip\{b_1 + 3b_3\} + L^2p^2(6a_1b_3 + 16a_2b_4 + 30a_3b_5 \cdots)] \\
&+ e^{-3ipt}[6BLip\,b_3 + LEip\{2b_2 + 4b_4\} + L^2p^2(8a_1b_4 + 2a_2b_5 + 36a_3b_6 - 4b_1b_2 \cdots)] \\
&+ e^{-4ipt}[8BLip\,b_4 + LEip\,(3b_3 + 5b_5) + L^2p^2(-4b_2^2 - 6b_1b_3 + 10a_1b_5 + 24a_2b_6 \cdots)] \\
&+ e^{-5ipt}[\cdots\cdots] \\
&\cdots\cdots
\end{aligned}
$$

Determination of the Coefficients.

Equating coefficients we obtain

$$
\begin{aligned}
a_0/2 &= B^2 + \frac{E^2}{2} - L^2p^2(2a_1b_1 + 4a_2b_2 + 18a_3b_3 + 32a_4b_4 \cdots) \\
a_1 &= -2BLip\,a_1 + BE - 2LEip\,a_2 + L^2p^2(4a_2b_1 + 12a_3b_2 + 24a_4b_3 + 40a_5b_4 \cdots) \\
a_2 &= -4BLip\,a_2 + \frac{E^2}{4} - LEip\,(a_1 + 3a_3) - L^2p^2(a_1^2 - 6a_3b_1 - 16a_4b_2 - 30a_5b_3 \cdots) \\
a_3 &= -6BLip\,a_3 - LEip\,(2a_2 + 4a_4) - L^2p^2(4a_1a_2 - 8a_4b_1 - 20a_5b_2 - 36a_6b_3 \cdots) \\
a_4 &= -8BLip\,a_4 - LEip\,(5a_5 + 3a_3) - L^2p^2(4a_2^2 + 6a_1a_3 - 10a_5b_1 - 24a_6b_2 \cdots) \\
a_5 &= \cdots\cdots \\
&\cdots\cdots
\end{aligned}
$$

$$b_1 \quad = 2\,B\,L\,ip\,b_1 + B\,E + 2\,L\,E\,ip\,b_2 + L^2p^2(4a_1b_2 + 12a_2b_3 + 24a_3b_4 + 40a_4b_5 \text{———})$$

$$b_2 \quad = 4\,B\,L\,ip\,b_2 + \frac{E^2}{4} + L\,E\,ip\,(b_1 + 3b_3) + L^2p^2(-b_1^2 + 6a_1b_3 + 16a_2b_4 + 30a_3b_5 \text{———})$$

$$b_3 \quad = 6\,B\,L\,ip\,b_3 + L\,E\,ip\,(2b_2 + 4b_4) + L^2p^2(-4b_1b_2 + 8b_1b_4 + 20a_2b_5 + 36a_3b_6 \text{———})$$

$$b_4 \quad = 8\,L\,ip\,b_4 + L\,E\,ip\,(3b_3 + 5b_5) + L^2p^2(-4b_2^2 - 6b_1b_3 + 10\,a_1b_5 + 24a_2b_6 \text{———})$$

b_5 = -

- -

In order to obtain a complete solution for the current in terms of all its constituent harmonics it would be necessary to solve an infinite number of equations for an infinite number of variables. As this is not within the limits of practicability, it is necessary to make some assumptions as to the nature of the coefficients. We saw in the case of the resistance in the plate circuit that the first few terms of the series were the only ones of importance for values of "e" within the limits (9). We also saw that values of "e" up to 10 or 15 volts, that the amplitute of the fundamental could be considered as varying directly as the first power of "e". It will be readily seen that if all the coefficients above, except a_0 , a_1 and b_1 , be considered negligible in comparison with these, that we shall obtain an expression for the fundamental amplitude in this case which will vary directly as the first power of the input grid voltage.

Neglecting all coefficients except a_0 , a_1 and b_1 , we obtain

$$a_1 = \frac{B\,E}{1 + 2BLip}$$

$$b_1 = \frac{B\,E}{1 - 2BLip} \qquad (24)$$

Substituting these values in the expression for a_2 and b_2 , we obtain for a_2 and b_2

$$a_2 = \frac{E}{4(1 + 2BLip)^2(1 + 4BLip)}$$

$$b_2 = \frac{E}{4(1 - 2BLip)^2(1 - 4BLip)} \qquad (25)$$

Substituting these values in the equations for a and b , and neglecting the coefficients with subscripts above three, the valúes of the latter may be determined, and using these values a_{1} and b_{1} may be obtained and so on for higher terms.

From relation (22) the values of $\alpha_1, \beta_1, \alpha_2, \beta_2$, etc., may be found.

$$\alpha_1 \sin pt + \beta_1 \cos pt = \frac{\mu e \cos (pt - \alpha)}{(R_o^2 + \ell^2 p^2)^{1/2}}, \text{ where } \alpha = \tan^{-1} \frac{\ell p}{R_o} \qquad (26)$$

$$\text{and } R_o = \frac{1}{2A(E_b + \mu E_g + \varepsilon)}$$

$$\alpha_2 \sin 2\,pt + \beta_2 \cos 2\,pt = \frac{A \mu^2 e^2 \cos (2\,pt - \beta)}{2(R_o^2 + \ell^2 p^2)(R_o^2 + 4\ell^2 p^2)^{1/2}} \qquad (27)$$

$$\text{where } \beta = \tan^{-1} \frac{2\ell p}{R_o} \left[\frac{2 R_o^2 - \ell^2 p^2}{R_o^2 - 5\,\ell^2 p^2}\right]$$

Similarly $\alpha_3 \sin 3\,pt + \beta_3 \cos 3\,pt$ may be found and so on for the higher terms.

Since $I = a_{o/2} + \Sigma \alpha_n \sin (n\,p\,t) + \Sigma \beta_n \cos n\,pt$, the addition of the varies quantities found above will give the resulting current I.

It is obvious that as soon as the values of a_2 and b_2 become appreciable compared with a_1 and b_1, the values of the latter obtained above can no longer be corrent, since they were determined on the basis that all the other coefficients were negligible.

If the values obtained for a_2 and b_2 are substituted in the equations for a_1 and b_1, the following is the value of

$$\alpha_1 \sin pt + \beta_1 \cos pt = \frac{\mu e \cos (pt - \alpha)}{(R_o^2 + \ell^2 p^2)^{1/2}}$$

$$+ \frac{2 R_o^5 A^2 \mu^3 e^3 \;\; \ell\, p \cos (pt - \epsilon)}{(R_o^2 + \ell^2 p^2)^2 (R_o^2 + 4\ell^2 p^2)^{1/2}} \qquad (28)$$

$$\text{where } \alpha = \tan^{-1} \frac{\ell p}{R_o}$$

$$\text{and } \epsilon = \tan^{-1} \frac{R_o}{2\ell p} \left(\frac{\ell^2 p^2 - 2 R_o^2}{R_o^2 - 5\,\ell^2 p^2}\right)$$

In order to obtain a numerical value for $\alpha_1 \sin (pt) + \beta_1 \cos pt$, it is best to evaluate each term separately and then compound the results by the parallelogram law. Similarly if the values of a_3 and b_3 become comparable with a_2 and b_2, we find for the corrected value of $\alpha_2 \sin 2\,pt + \beta_2 \cos 3\,pt$

$$= \frac{R_o^3\, A \mu^2 e^2 \cos(2\,pt - \beta)}{2(R_o^2 + \ell^2 p^2)(R_o^2 + 4\ell^2 p^2)^{1/2}} + \frac{12\, R_o^7\, A^3 \mu^4 e^4\, \ell^2 p^2 \cos(2\,pt - \lambda)}{(R_o^2 + \ell^2 p^2)^2 (R_o^2 + 4\ell^2 p^2)(R_o^2 + 9\ell^2 p^2)^{1/2}}$$

where β is the same as defined in (27) and

$$\lambda = \tan^{-1} \frac{3\ell p}{R_o} \left(\frac{3\,R_o^4 - 17\,\ell^2 p^2 R_o^2 - 8\,\ell^4 p^4}{R_o^4 - 32\,\ell^2 p^2\, R_o + 40\,\ell^4 p^4} \right)$$

By making successive approximations as many terms as desired may be included in the expressions for any particular harmonic. It will be noticed that the resulting angle of lag of each harmonic depends on the number of terms we include in the coefficient, and thus there arises a peculiarity in a vacuum tube generator, namely that the angles of lag of the various output harmonics are dependent on the amplitude of the input wave on the grid. The larger the amplitude for the coefficients of the various harmonics, and the consequent shifting of the angles of lag results.

Area of the Characteristic Loop

Since the fundamental plate current lags behind the grid voltage by an angle $\alpha = \tan^{-1} \frac{\ell p}{R}$, it is evident that if this current be plotted against the alternating grid voltage that an elliptical characteristic will be produced. If, however, the first and higher harmonics are plotted in addition to the fundamental, and the curves thus formed compounded into a single curve, it is evident that the characteristic will no longer be a true ellipse. Since the amplitudes of the harmonics are small compared to that of the fundamental, the resulting curve will not seriously depart from an ellipse. This curve is what is usually referred to as the dynamic characteristic. The $a_o/2$ term of the series gives the point of operation on the static characteristic, and it is obvious from the expression for the latter, that the larger the harmonics become, the greater is the shift of this point of operation. In practice this shift is noticed by the increased reading of a direct current milliameter.

If all but the fundamental current is omitted, the area of the loop may be easily found.

Put $x = E_c + e \cos pt$

$$y = a_0/_2 + \frac{\mu e}{(R_0^2 + \ell^2 p^2)^{1/2}} \cos (pt - \alpha)$$

Limits for pt are o and 2π

$$\text{Area} = \int_0^{2\pi} ydx = -\int_0^{2\pi} \left\{ \frac{a_0}{2} + \frac{\mu e}{(R_0^2 + \ell^2 p^2)^{1/2}} \cos (pt - \alpha) \right\} \sin pt\, dt$$

$$= \frac{2\pi e^2 \mu \sin \alpha}{(R_0^2 + \ell^2 p^2)^{1/2}}$$

$$= \frac{2\pi \mu e^2 \ell p}{(R_0^2 + \ell^2 p^2)} \quad \text{since} \quad \alpha = \tan^{-1} \frac{\ell p}{R_0}$$

If the curve be referred to the I_b, E_b axes, this expression must be multiplied by . Also since the maximum value I_0 of the fundamental is $\frac{\mu e}{(R_0^2 + \ell^2 p^2)^{1/2}}$ the area of the loop may be written as

$$A = 2\pi I_0^2 \ell p = 2\pi \frac{\ell p}{R_0} R_0 I_0^2$$

Since $R_0 I_0^2$ = the power dissipated in the plate circuit the area of the loop is thus proportional to that power. If the inductance were not present A = o, which is the same thing as saying that the characteristic no longer has the form of a loop, but reverts back to the type found when a resistance was placed in the plate circuit.

<u>Case of a capacity in the plate circuit.</u>

Since a condenser placed between the plate and the battery prevents the direct current from flowing, it is necessary to place a choke coil across the condenser. The choke coil will allow the direct current to pass, but if made properly will offer almost an infinite resistance to the high frequency current.

The solution for this case is directly analagous to that for the inductance problem. The only difference in the final result being that $\frac{1}{cp}$ always replaces ℓp.

Thus the simple expression for the fundamental becomes $\frac{\mu e}{(R_0^2 + \frac{1}{c^2p^2})^{1/2}} \cos(pt - \alpha_1)$ where $\alpha_1 = \tan^{-1} \frac{1}{R_0 c p}$. Similar expressions for the other harmonics may be found from comparison with the expressions found for the inductance.

The dynamic characteristic for this case is similar to that found for the inductance, the only difference being that it is traced out in the opposite direction.

Description of Apparatus and Experimental Procedure

In order to have an experimental set-up which could be used to verify the preceding theory, the following conditions and requirements had to be met.

(a) Production of a pure sine wave e. m. f.

(b) Use of a sufficiently low frequency that capacity effects might be of small dimensions.

(c) Accurate measurement of the input e.m.f. on the grid of the harmonic producing tube.

(d) Use of a pure resistance.

(e) Use of a pure inductance.

(f) Measurement of the amplitude of the harmonics produced, without introducing extraneous resistances, etc. into the harmonic producer.

Figure VII is the complete circuit diagram of the entire collection of apparatus used in the experimental work. It may be divided into three main sections, the oscillator, harmonic producer and the harmonic analyser. The oscillator in the upper left corner is designed so as to produce as pure a sine wave as possible. The tuned circuit $L_1 C_1$ prevents the fundamental frequency from passing into the battery circuit, thus compelling it to travel to the filament through the inductance L_3 of the oscillating circuit. The condenser C_1 offers less and less impedance to the higher harmonics, and the latter will pass down through C_1 to the filament terminal. As the first harmonic is always appreciable, it was specially filtered out of the oscillating circuit $L_3 C_3$, by means of the anti-resonant circuit $L_2 C_2$. The inductance L_2 =.52 M.H., of course offered some impedance to the fundamental frequency, but that was negligible compared with the impedance that $L_1 C_1$ offered to the fundamental. These filters thus helped to produce a pure sine wave oscillation of the same frequency as the fundamental in the circuit L_3 C_3. The frequency used throughout was 200,000 cycles per sec., or a wave length of 1500 meters. This frequency was high enough that it could be tuned very sharply, and yet not so high that the internal capacities of the tubes would be of any importance. Ballentine[1] has worked out expressions for the input impedance of tubes under various conditions, and applying his formulae to the W.E. 205 B tube at this frequency and under the experimental conditions which will be described below, the input impedance was of the order of 100,000 ohms.

The inductance L_4 was loosely coupled to L_3 and connected by means of a twisted pair with L_5, which in turn was loosely coupled to L_6. These latter coils were placed about seven meters away from the oscillating circuit, in order that they might not pick up any of the harmonics. The loosening of the couplers already described resulted in maintenance of the sine e. m. f. The combination of condensers C_4, C_5, C_6 and C_7 and the inductance L_6 is tuned for the fundamental frequency. The arrangement of these condensers is what is known as a potential divider and has been described by Hulbert and Breit.[2] The object is to take a portion of the alternating e. m. f. across L_6 and impress it on the grid of a tube. When the thermocouple is in the dotted position the current passing through C_4 is measured, but the current passing through C_5 and C_6 can easily be calculated when the values of the different capacities are known. The object of measuring the current in C_4 is that for small values of input potentials, the currents passing through C_5 and C_6 would be too small to be recorded by a low resistance thermo-couple. If I represents the amplitude of the alternating current passing through C_5 and C_6 then the resulting input e. m. f. is $\frac{I}{2\pi(C_5 + C_6) f}$, where f is the frequency of the wave. For an e. m. f. of over one volt, the current I could be measured directly by the low resistance thermo-couple.

The resistance R_2 is used to provide a leak for any charge that my accumulate on the grid, and allow it to flow to earth. Its resistance must be comparable to the input impedance of the tube. A dilute solution of copper sulphate in water worked satisfactorily as a high resistance. By means of potentiometer R_3

1. loc. cit

2. Phys. Rev. 4, 278. 1920

the potential on the grid could be varied as desired.

The upper tube to the right is the harmonic producer. By means of the condenser potential divider a known value of input e. m. f. was impressed between the grid and filament and then according to equations (11), for a resistance E F in the plate circuit, and (19) for an inductance E F, a plate current will result which is capable of being represented as a series of harmonics. In order to get the results predicted in equation (11), E F must be a pure resistance. A straight wire immediately suggests itself as a resistance which would possess a minimum inductance and capacity. However, in order to obtain a resistance of the order of 3000 ohms, so much wire would be needed, that inductive and capacitive effects would become appreciable. Then again, it is a well known fact the conductivity of a wire diminishes with the frequency owing to the skin effect, and consequently the exact value of the resistance at any particular frequency is not easily determined. A resistance suitable for high frequency work should have a negligible skin effect, as well as having negligible inductance and capacity. On the suggestion of Professor W. F. G. Swann, the author tried out some platinised quarts fibres immersed in acid-free paraffin oil, and found that they would carry currents up to at least 60 milamperes. From the formula for change in resistance with frequency[1], it can be shown that using fibres about .01 M. M. in diameter, the skin effect can be neglected. As the fibres used had a resistance of about 100 ohms per cm. only a short length of circuit was needed, thus reducing the inductance and capacity. The inductance E F was wound with No. 16, D. C. C. copper wire, and the windings were spaced about 1 m. m. apart. The resistance of the coil was 10 ohms, whereas the reactance was 2700 ohms.

In order to detect the various harmonics, a fraction of the e. m. f. along E F was impressed on the grid of a W. E-D tube, this impressed e. m. f. being always kept less than 1 volt. A 100 Ohm slide wire of I_a I_a wire was used for the

1. Fleming. Wireless Telegraphy.

variable portion of E F. It can be seen from equation (18) that in order to obtain pure amplification without the introduction of harmonics whose amplitudes are appreciable compared to that of the fundamental, the input e. m. f. must be small, (less than one volt) and the value of the external resistance must be high. The resistance of an anti-resonant circuit is $R + \frac{L^2\omega^2}{R}$, where $L\omega$ is the inductive reactance. Since R, the ohmic resistance, is negligble, at radio frequencies, in comparison with $\frac{L^2\omega^2}{R}$, the latter term may be taken as the value of the resistance of an anti-resonant circuit. Therefore for any given ω, L should be made as large as possible and R as small as possible. Now in the plate circuit of the tube which is used to separate out the harmonics, a series of anti-resonant circuits are placed, The first one is tuned for the fundamental, the second for the first harmonic, and so on. A vacuum thermocouple is placed in each circuit on the capacity side. This is done so that the D. C. plate current will not affect it. In order to keep the ohmic resistance low, thermocouples with heater resistances of from .5 ohm to 5 ohms were used. The higher resistance thermocouples being used to measure the weaker amplitudes of the higher harmonics.

The effective resistance of these circuits are as follows:

Fundamental	120,000 ohms.
First harmonic	183,000 ohms.
Second harmonic	95,000 ohms.
Third harmonic	175,000 ohms.

Arrangements were also made (not shown in Figure II), for measuring higher harmonics than these, by changing the inductance $L_{\prime\prime}$, and by retuning $C_{\prime\prime}$. Each thermo-couple could be connected successively to a Lees and Northrup Galvanometer, and the deflection of the latter indicated the root-mean square value of the alternating current passing through the heater. Previous to placing the thermo-couples in the circuits, they were calibrated using alternating current (60 cycles). It will be seen that the harmonic analyser is essentially a voltage amplifier, picking out each frequency in the producer and magnifying its voltage. For this reason a tube with a large voltage amplification constant was chosen, in fact the value of " μ " as given by equation (4) was 26.

Let I_1 = the maximum current in the Fundamental circuit.

Let R_1 = the effective resistance.

Let R_0 = the internal output resistance of the D tube .

Therefore $R_1 I_1$ = e. m. f. across L_f.

Let e = e. m. f. across F G

Let μ' = actual voltage amplification factor

$$\text{Therefore } \mu' = \frac{\mu R_1}{R_0 + R_1} = \frac{26 \times 120{,}000}{29{,}250 + 120{,}000} = 20.8$$

$$\text{Therefore } e = \frac{R_1 I_1}{20.8} = \frac{120{,}000 \times I_1}{20.8}$$

Let r = resistance of F G

Let i = amplitude of current of fundamental frequency passing through F G.

$$\text{Therefore } r\,i = e \text{ and } i = \frac{120{,}000.\ I_1}{20.8 \times r} \tag{25}$$

Thus knowing I_1 from the galvanometer deflection, and r from the Wheatstone Bridge, the value of i can be determined. For the case worked out above " i", represents the amplitude of the fundamental frequency produced by a pure sine wave impressed on the grid of a tube having a pure resistance load in the plate circuit. Similarly, by measuring the currents in the other tuned circuits we can work back to the equivalent current in the harmonic producer.

When the resistance E F is replaced by an inductance, a portion F G of the inductance is used to obtain the input on the grid of the analyser. In this case it will be noted that E F offers twice as much impedance to the first harmonic, three times as much to the second harmonic, and so on. This makes it possible to measure weaker harmonics than in the case of the resistance. The inductance of G F in this experiment was .0587 mil-henries, whereas the whole inductance of E F was 2.14 mil-henries. The input impedance of the analyser to which E F was attached was of the order of 50,000 ohms at 200,000 cycles per sec., whereas the impedance of .0587 henries is only 74 ohms. This shows that the impedance of the analyser was practically short circuited by the coil F G, and consequently did not affect the nature of the external circuit of the producer. For measurement of large output current values, the value of F G was reduced to .04 M.H. To obtain, say the amplitude of the fundament current with the inductance, we have an equation analagous to (25).

$$i = \frac{120,000\ I}{20.8\ (\ell\ \omega)}$$, where ℓ is the inductance of F G and ω = $2\pi\ X$ the frequency.

Experimental Results.

The following constants for the 205 B tube were determined from its static characteristic.

$A = .554 \quad 10^{-6}$

$\epsilon = 7.5$ volts

$\mu = 6.7$

For $E_b = 260$ volts, $E_c = -7.5$ volts

$$R_o = \frac{1}{2A(E_b + \mu E_c + e)} = 3,570 \text{ ohms.}$$

Resistance in Plate Circuit.

When a resistance of 2700 ohms was placed in series with the plate, the value of E_b reduced to 200 volts, and the plate current then was 21.5 milamperes. Using these values of potential and resistance the curves shown in Figure III were obtained. The amplitude of the input e. m. f. was varied from 0 up to 50 volts. The latter maximum loaded the tube rather heavily, and the larger currents were maintained just long enough to obtain the necessary readings.

Keeping the e. m. f. of the input at 15 volts, the actual plate potential at 200 volts, and varying the static grid potential the curves in Figure IV were obtained, for ranges of E_c between -30 and +12 volts.

With a static voltage of - 7.5 on the grid and the alternating e. m. f. kept at 15 volts, the variation of the harmonics with plate voltage was determined, as in Figure V .

Figure VI represents the wave-shape of the plate current obtained when a pure sine wave e. m. f. of 15 volts is impressed on the grid, the plate and grid potentials being the same as stated above. The fundamental and first harmonic are the only components included in the wave-shape. The amplitudes of the other harmonics are too small to be shown on the same scale. Figure VI shows the dynamic characteristic for this case, where the wave shape thus obtained is plotted against the alternating input voltage.

In Figure VII the variation of the harmonics with the value of the external resistance is shown.

Inductance in Plate Circuit.

Exactly similar experimental procedure was undertaken for the inductances. A higher plate voltage and plate current was used here, since there were no delicate platinised quartz fibres to be dealt with. For this case

$$E_b = 250 \text{ volts.}$$

$$E_c = -10 \text{ volts.}$$

$$R_o = \frac{1}{2A(E_b + \mu E_c + e)} = 3800 \text{ ohms.}$$

The reactive load in the plate circuit was 2700 ohms. Figure VIII shows the variation of the harmonics with the alternating e. m. f. on the grid, under the conditions given above.

In Figure IX is shown the variation of the harmonics with the value of the static grid potential, the alternating input e. m. f. being constant at 20 volts, and the plate voltage being 250 volts. Figure X shows how the variation of the static plate voltage affects the value of the harmonics.

The curves of Figure XI give the variation of the harmonics with the magnitude of the inductance in the plate circuit.

Figure XII represents the wave shape, using only the values of the fundamental and first harmonic.

Figure XII represents the dynamic characterictis for this case, the area of this loop being proportional to the amount of energy delivered.

Discussion of Results

The theoretical curves as shown in the various figures, are plotted from the values of the coefficients given by equations (10) and (18). In the case of the resistance the experimental fundamental values check up very closely with the theoretical values up to an input voltage of fifteen volts. Beyond that voltage equation (10) no longer accurately represents conditions. It will be noticed that the maximum positive potential to which the grid is raised in this operation is 7.5 volts. It will be seen from figure VI that at this voltage the static characteristic begins to flatten out, due to the passage of electrons to the grid instead of to the plate. For voltages higher than fifteen it will be noticed that the theoretical first and third harmonics fall below the experimental values, whereas the second harmonic appears to be greater than the experimental values would indicate. This would seem to be due to the flattening out of the wave at the upper end of the characteristic. In the case of the inductance good agreement between theoretical and experimental values were found for input voltages as high as 20 volts. This is due to the fact in this case that the operation was carried out over the 250 volt static characteristic, and it will be seen from figure XII that this curve does not begin to flatten out until a positive grid voltage of ten volts is reached. This point of operation coincides with the maximum positive voltage to which the grid was raised with an input e. m. f. of 20 volts, and E_c being - 10 volts.

The curves of figure IV & IX show, in an emphatic manner, that the further we move towards the straighter portion of the static characteristic, the greater the fundamental becomes while the other harmonics continue to diminish. At -40 volts the higher harmonics were still very much in evidence although the fundamental was rapidly approaching zero. The second harmonic in the resistance curves and the third in the inductance curves show rather peculiar irregularities. The sharp maxima and minima would at first sight seem to point to some sort of internal resonance in the tube. It will be noticed, however, that these are produced by simply varying either the grid or plate potentials, and are probably due to irregularities in the static characteristic which are smoothed over in the ordinary process of plotting. It will be recalled that in the case of the resistance the coefficient of the second harmonic is principally determined by the value of the third derivative of the particular function. A small and almost unnoticed irregularity in the static characteristic might produce a very large variation in the third derivative which, of course, would appear in its magnified form in the method described above for separating out each harmonic. In the case of the inductance the coefficients cannot be expressed as simple derivatives, but doubtless the reasoning followed above would suffice to explain the sharp variations in the third harmonic.

The curves of figures V & X show that the greater the plate potential the greater is the value of the fundamental while the harmonics continue to diminish. Beyond a certain plate potential, about 300 volts the fundamental became nearly constant in value. The curves which exhibited the irregularities in the grid-variation series, also exhibit similar irregularities here. Further, these irregularities occur at the same value of ($E_b + \mu E_c$) showing that the effect is independent of whether the grid or plate potential is increased provided that the sum as given is the same.

The curves of figures VII and XI show how the harmonics depend on the external impedance of the plate circuit. From these curves it would seem that the harmonics all approach a maximum value for zero impedance. Equation (9), however, says that for zero impedance in the external plate circuit, all harmonics above the first are zero. Of course, this latter condition can never be realised in practice for impedance of some sort must be introduced in order to measure the various harmonics. It would seem that for any small increase of external impedance above zero value the plate current harmonics must increase very rapidly, reaching a maximum after a few ohms has been added to the circuit. The only other alternative would be for the harmonics to suffer a finite discontinuity when the condition for zero impedance was reached. This, however, does not appear to be possible from physical considerations.

If all the harmonics were neglected the wave shape for the resistance would be a pure sine wave in phase with the impressed e. m. f. These two when compounded would give a straight line dynamic characteristic. It may be seen that with a sufficiently high resistance in the plate this condition may be nearly reached. However, if the higher harmonics were also taken into consideration , the dynamic characteristic is no longer a straight line, but has a curvature, which is much less that that of the static characteristic curve. With the inductance, if all but the fundamental had been neglected, the current wave would have been a pure sine wave lagging behind the impressed e. m. f. by an angle $\theta = \tan^{-1}\frac{\ell p}{R_o}$. The dynamic characteristic under those conditions would have been an ellipse. Addition of the other harmonic components, tends to flatten out the sine wave, and consequently distort the purely elliptical dynamic characteristic.

In calculating the theoretical values of the coefficients of the various harmonics, it might be noted that up to about 15 or 20 volts, the coefficients of the fundamental was practically proportional to the first power of the applied e. m. f., the first harmonic was proportional the the second power and so on. It was only for values of input voltage beyond 20 volts that the more complicated expressions for the coefficients had to be evaluated.

Summary.

The Van der Bijl relation is sufficiently accurate to give a close approximation to the first three or four harmonics present in the plate circuit when a pure sine wave e. m. f. is impressed on the grid. The maximum value which this e. m. f. may have and still retain the above relation depends on the values of the steady grid and plate voltages used.

When no impedance is present in the plate circuit the dynamic and static characteristics are identical as far as the range of the former exists, but when an impedance is present these two curves differ, the amount of difference being a function of the value of the impedance.

The condition for maximum output when a resistance is placed in the plate circuit shows that an electron tube differs from an ordinary generator of alternating current in the fact that its internal resistance is a function of its external resistance and the static potentials.

In conclusion the writer wishes to express his very sincere thanks to the Western Electric Company Inc. of New York, for their kindness in loaning necessary tubes and vacuum thermo-couples, and also to Professor W. F. G. Swann of this department for his many helpful suggestions and criticisms and his invaluable encouragement at all times.

John G. Frayne
University of Minnesota
April 11, 1921.

JGF*VT

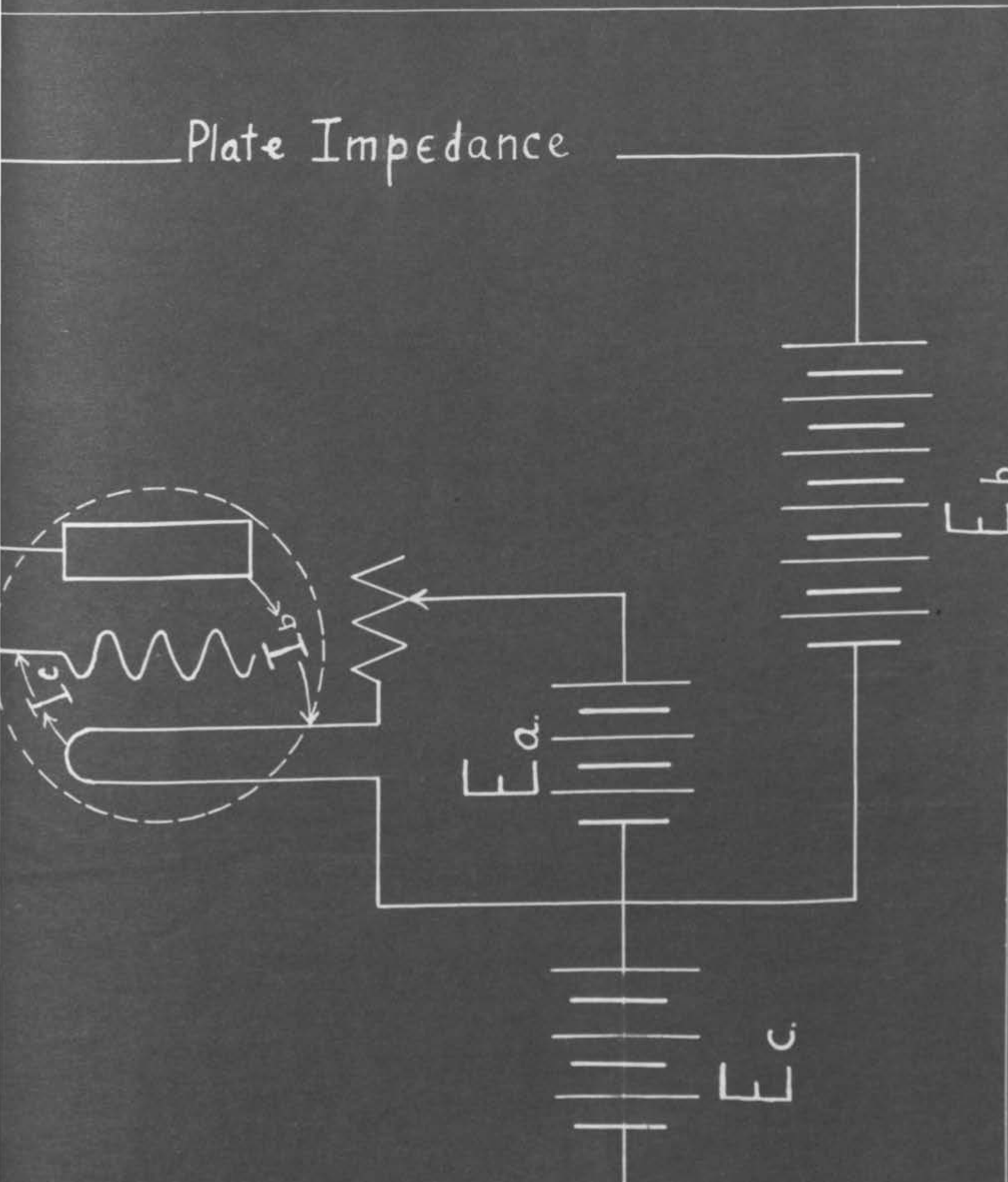
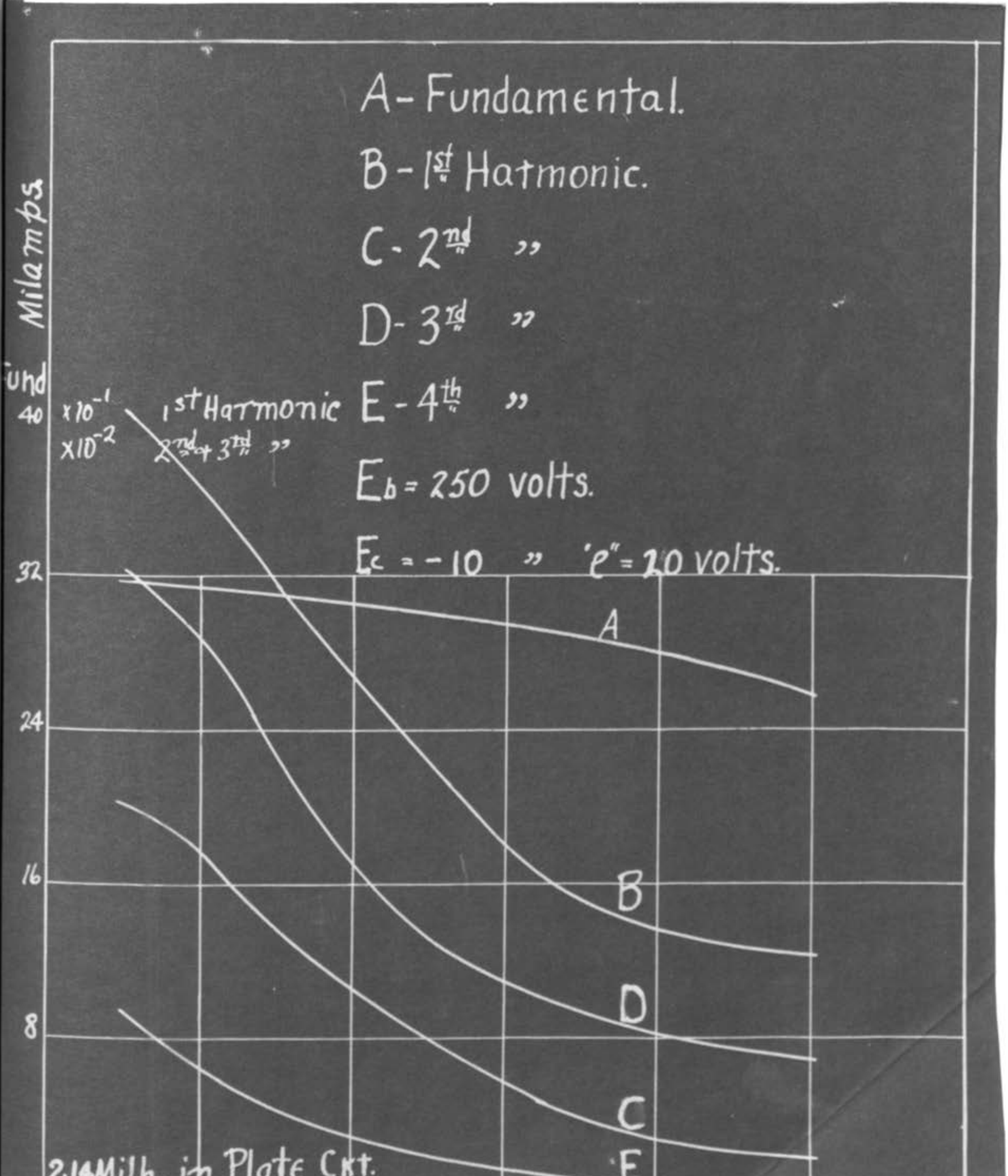


Fig. I.


A- Fundamental.
B - 1st Hatmonic.
C - 2nd "
D - 3rd "
E - 4th "
Eb = 250 volts.
Ec = -10 " "e" = 20 volts.
Milamps.
Fund
x10⁻¹ 1st Harmonic
x10⁻² 2nd & 3rd "
40
32
24
16
8
A
B
D
C
E
2.14Milh. in Plate Ckt.
·4
·8
1·6
2·0
2·4
Fig. 11.

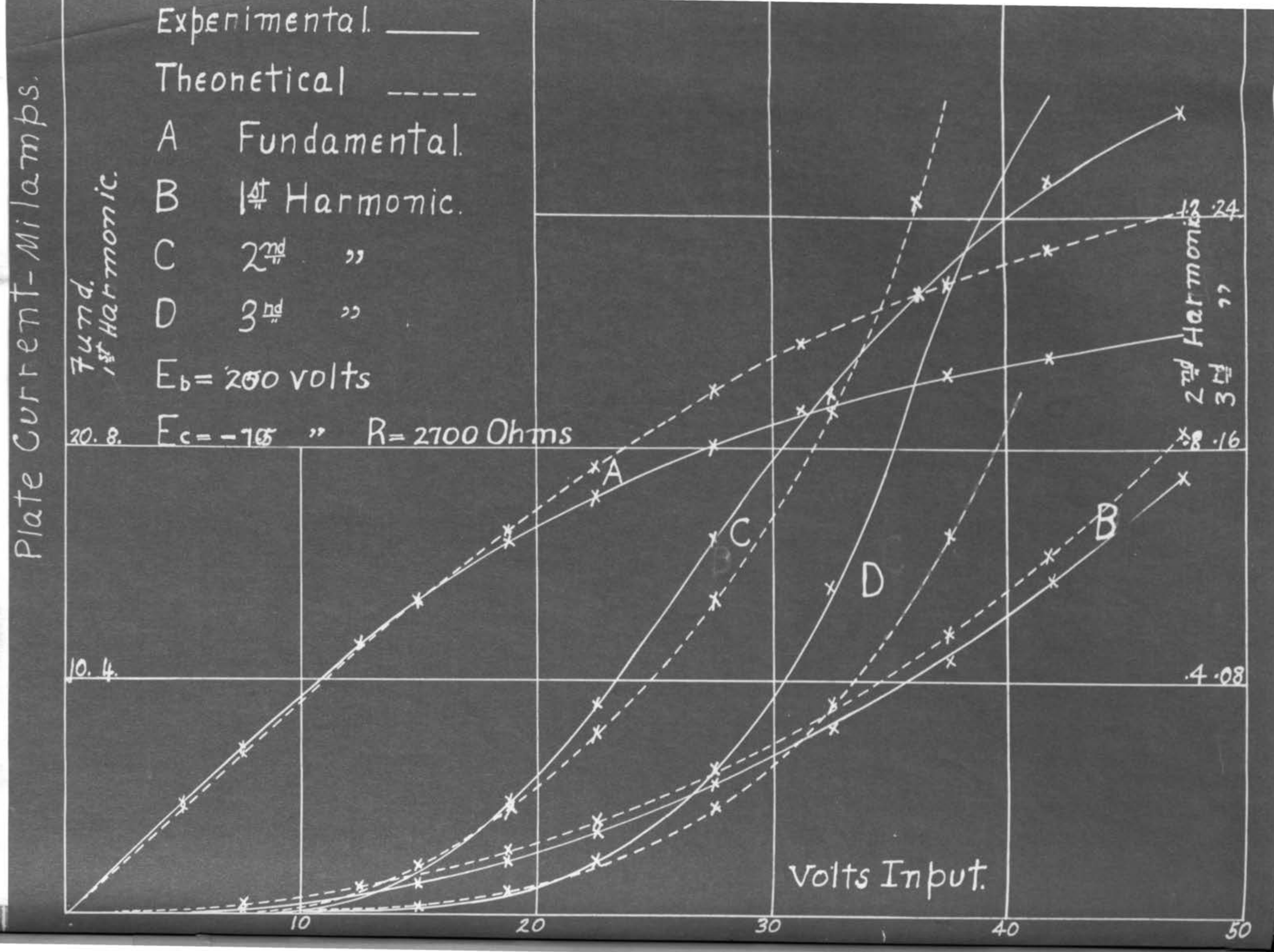


Plate Current- Milamps.
Fund.
1st Harmonic.
Experimental. ——
Theoretical -----
A Fundamental.
B 1st Harmonic.
C 2nd "
D 3rd "
Eb= 200 volts
Ec= -15 " R= 2700 Ohms
20. 8.
10. 4.
A
B
C
D
Volts Input.
10
20
30
40
50
2nd Harmonic
3rd "
.12 .24
.8 .16
.4 .08

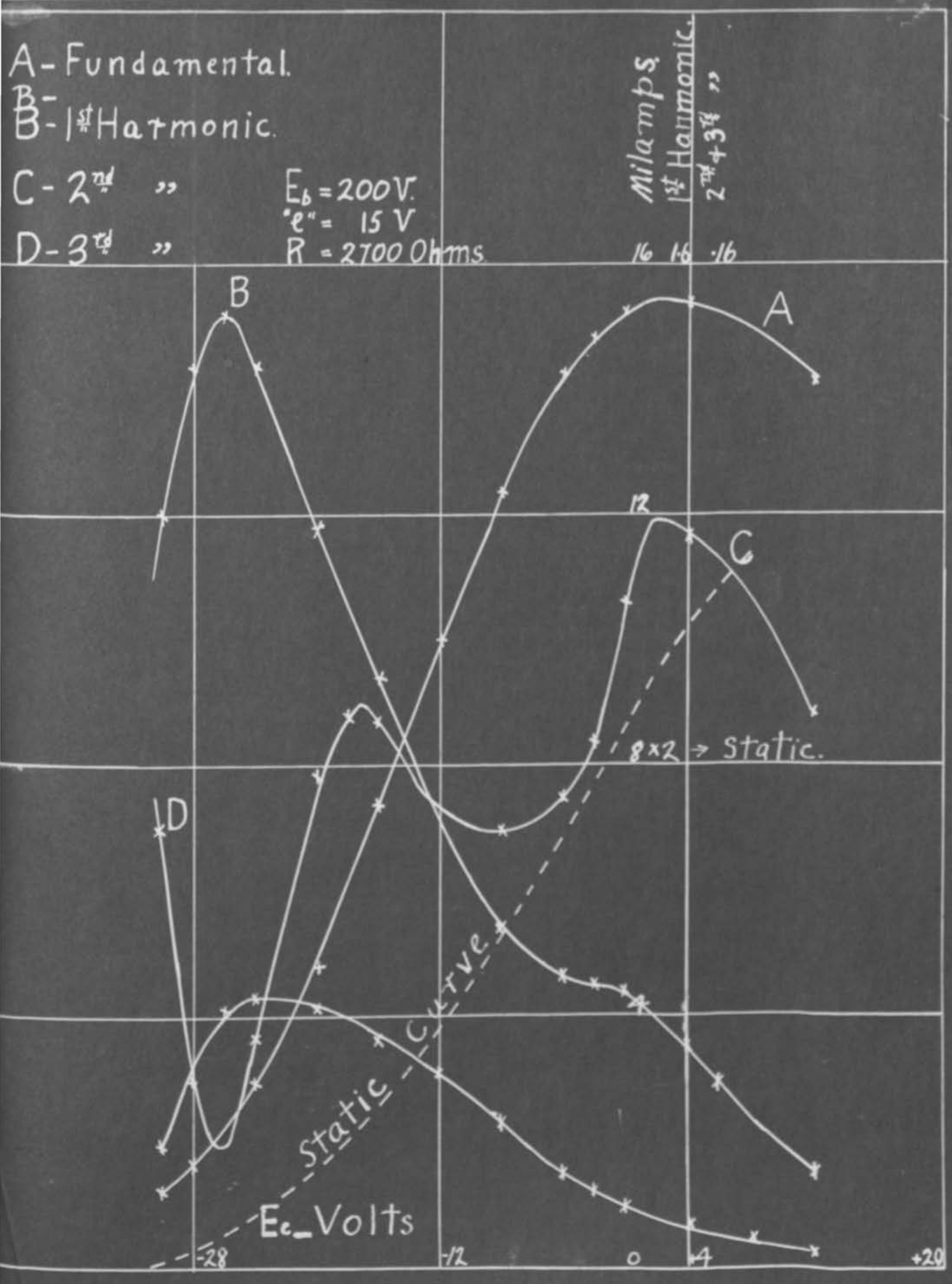

A- Fundamental.
B- 1st Harmonic.
C- 2nd ,,
D- 3rd ,,
E_b = 200V.
"e" = 15 V
R = 2700 Ohms
Milamps
1st Harmonic.
2nd + 3rd ,,
16 1.6 .16
A
B
C
D
12
8×2 → Static.
Static Curve
E_c Volts
-28
-12
0
+4
+20

Fig. 4

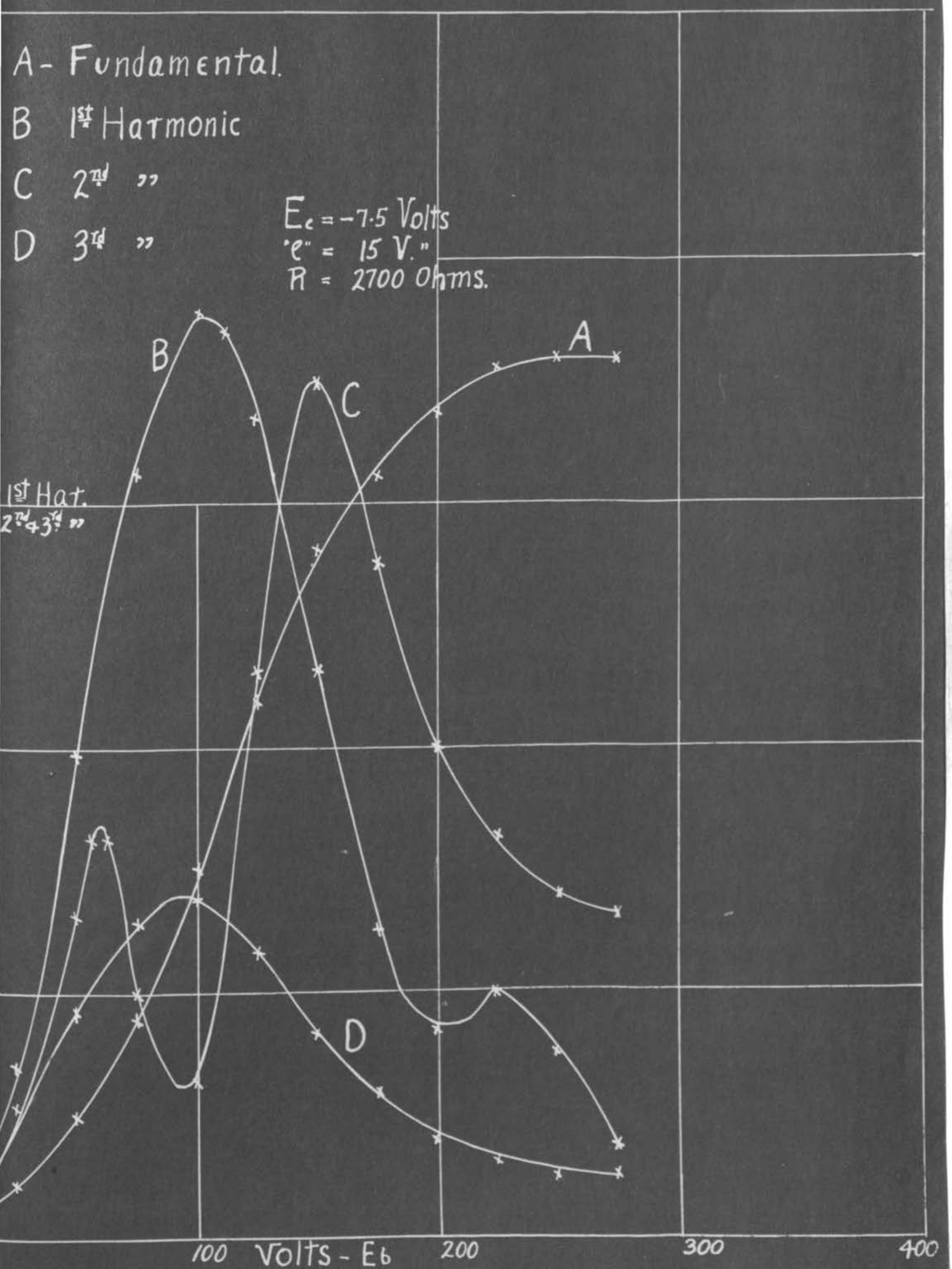

A- Fundamental.
B 1st Harmonic
C 2nd "
D 3rd "
Ec = -7.5 Volts
"e" = 15 V."
R = 2700 Ohms.
A
B
C
D
1st Har.
2nd & 3rd "
100
200
300
400
Volts - Eb

Fig. 5

Plate Current

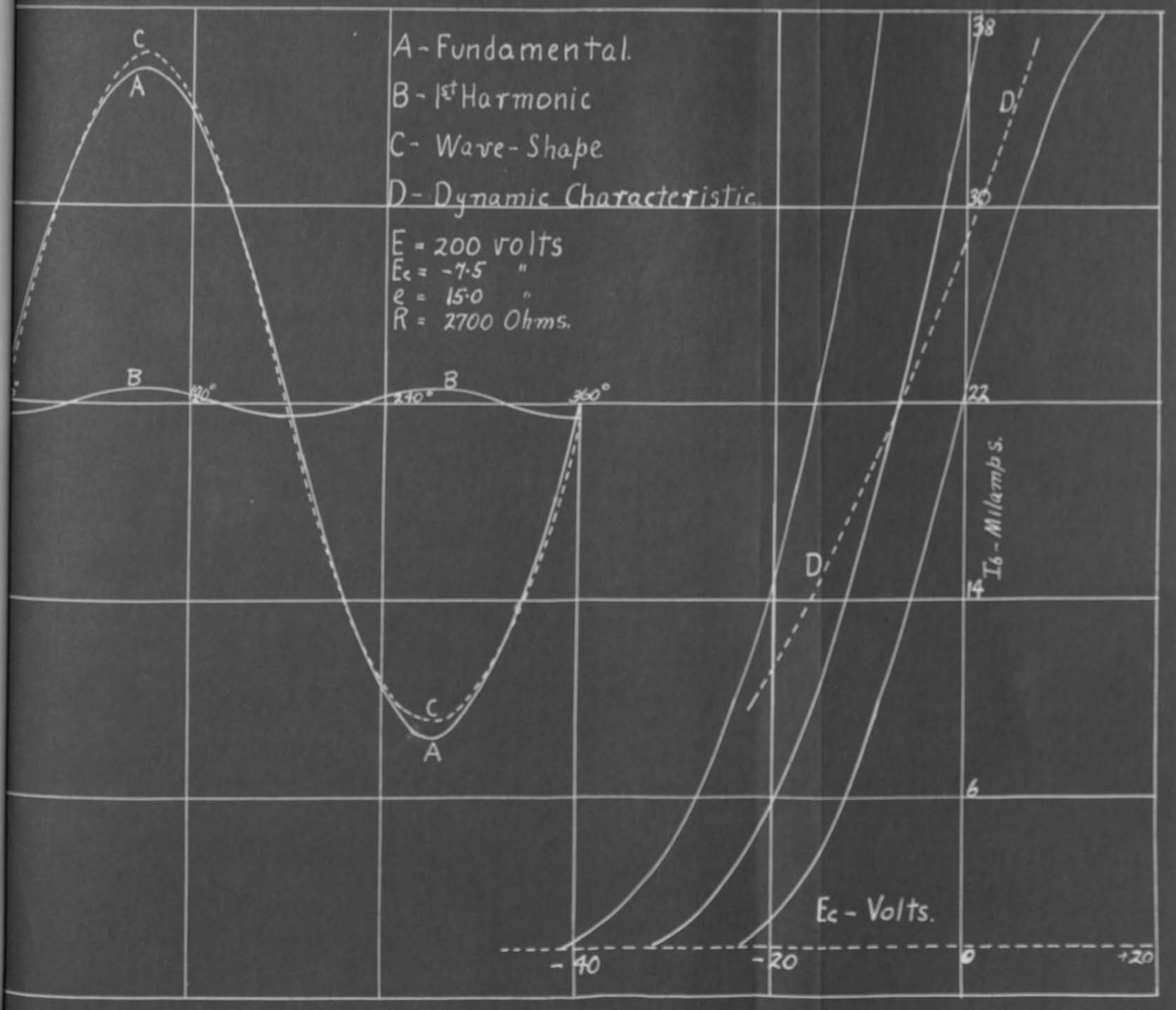


Fig. - 6

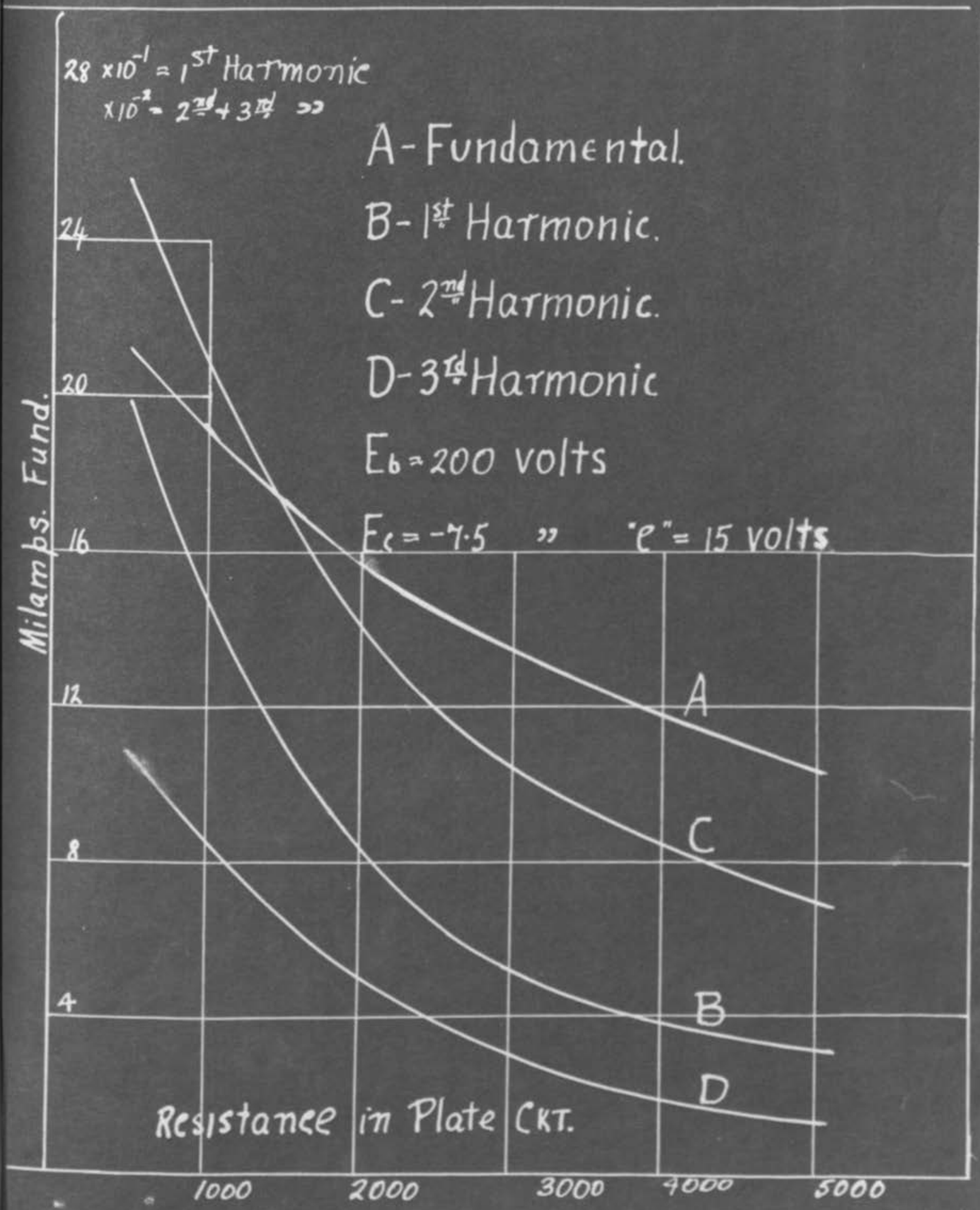

28 x10^{-1} = 1st Harmonic
x10^{-2} = 2nd + 3rd "
A- Fundamental.
B- 1st Harmonic.
C- 2nd Harmonic.
D- 3rd Harmonic
Eb = 200 volts
Ec = -7.5 " "e" = 15 volts
Milambs. Fund.
24
20
16
12
8
4
A
C
B
D
Resistance in Plate CKT.
1000
2000
3000
4000
5000

Fig. 7.

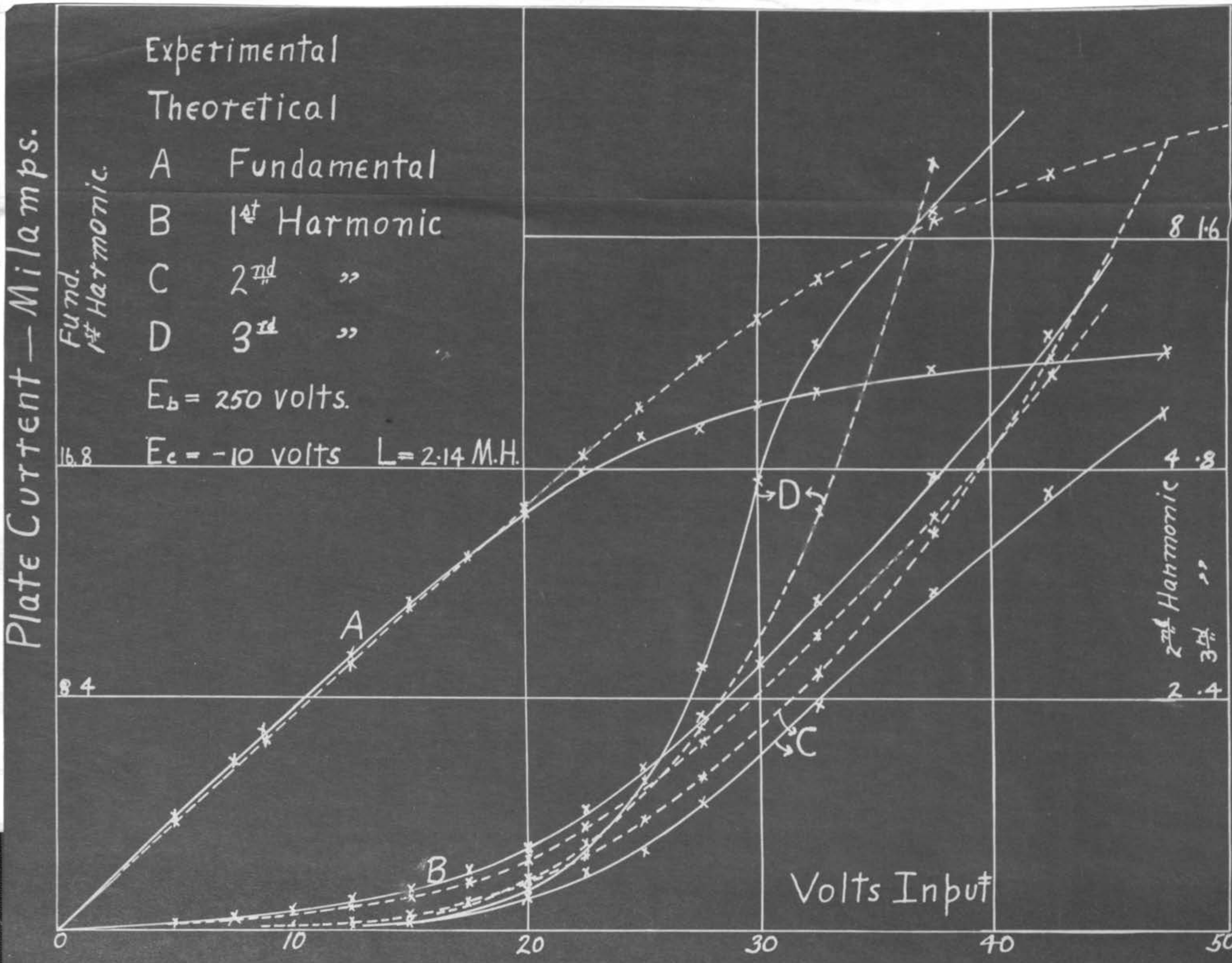
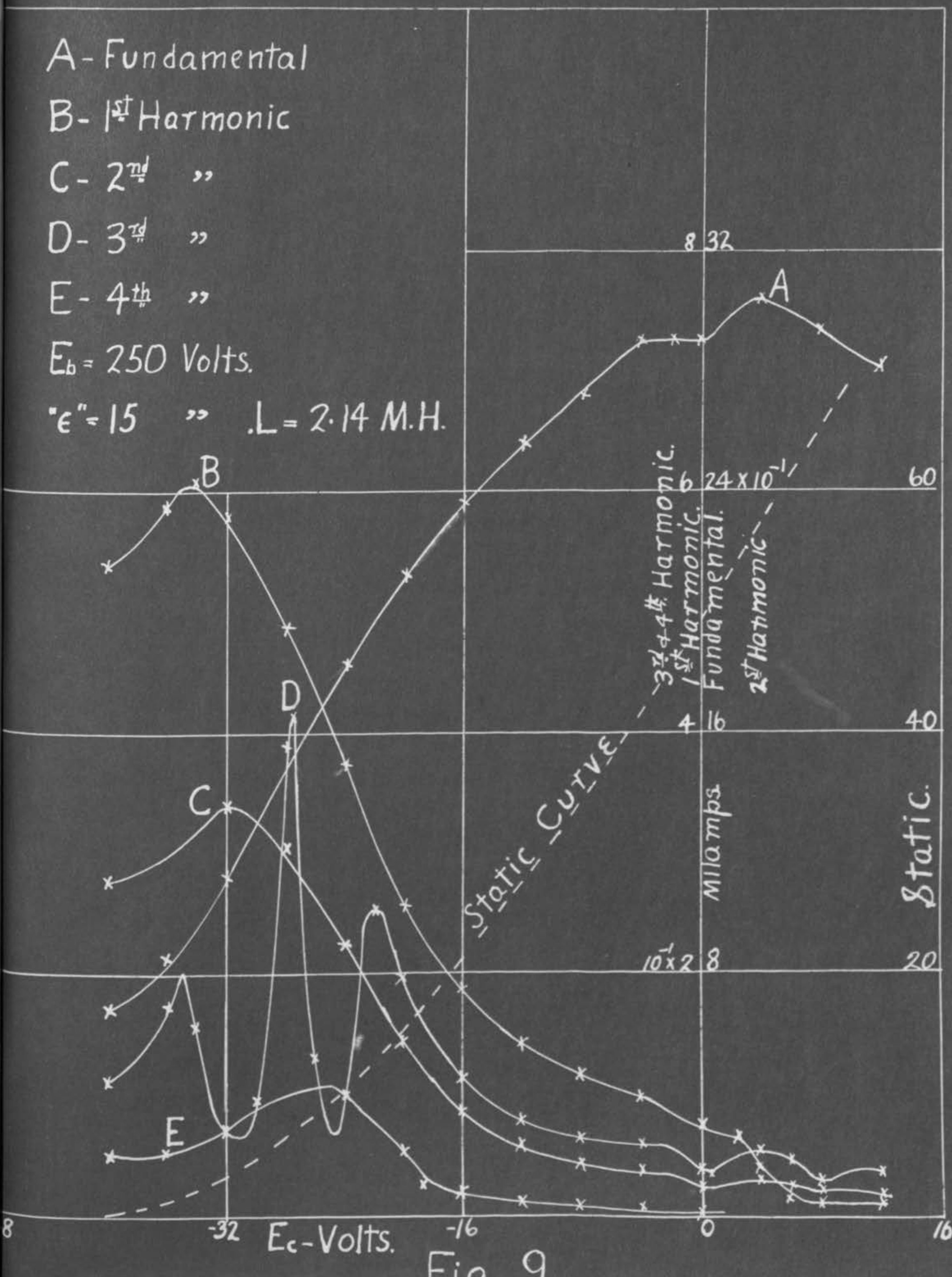


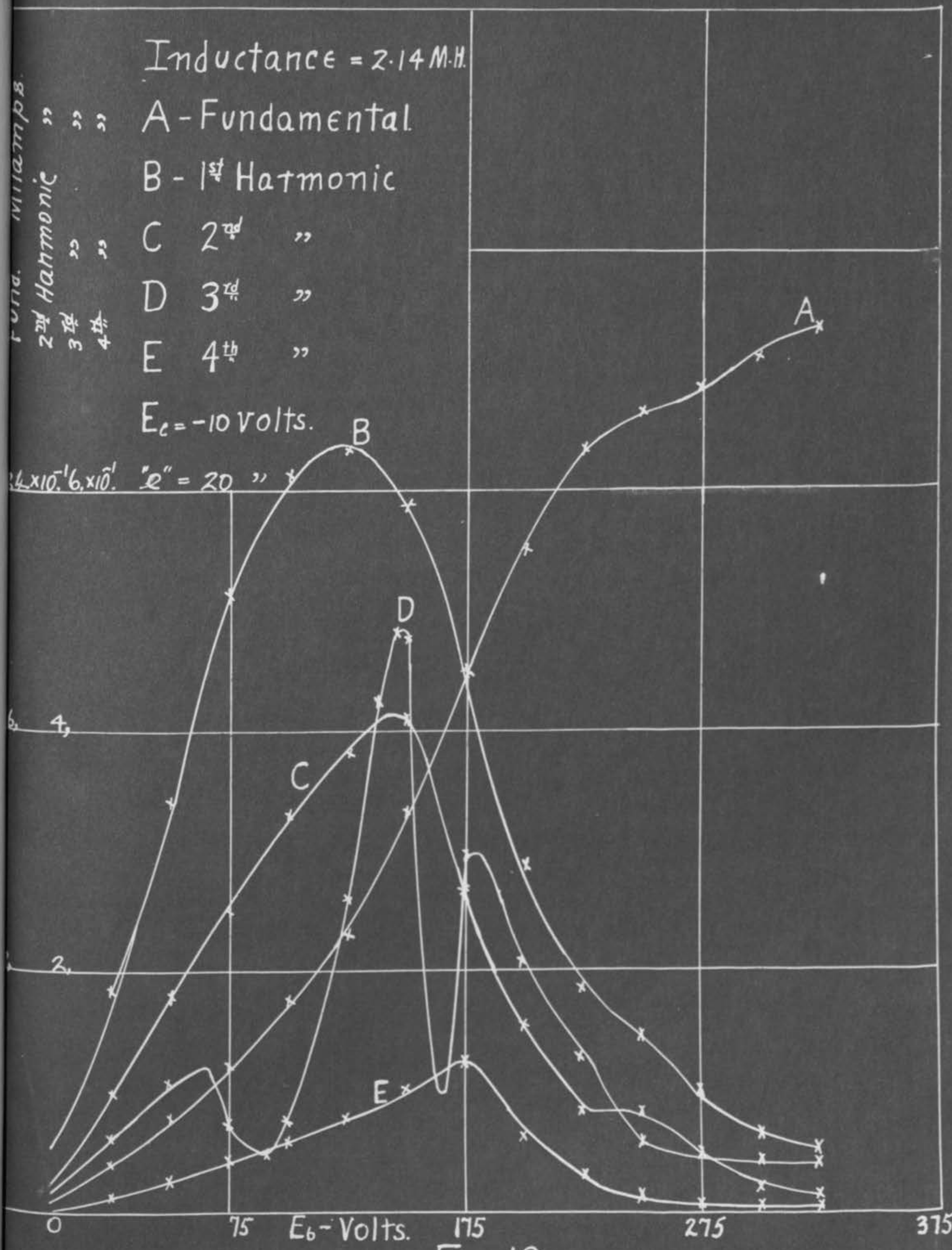
Plate Current — Milamps.
Fund.
1st Harmonic.
Experimental
Theoretical
A Fundamental
B 1st Harmonic
C 2nd "
D 3rd "
Eb = 250 volts.
Ec = -10 volts L = 2.14 M.H.
16.8
8.4
8 1.6
4 .8
2 .4
2nd Harmonic
3rd "
A
B
C
D
0
10
20
30
40
50
Volts Input

Fig. 8



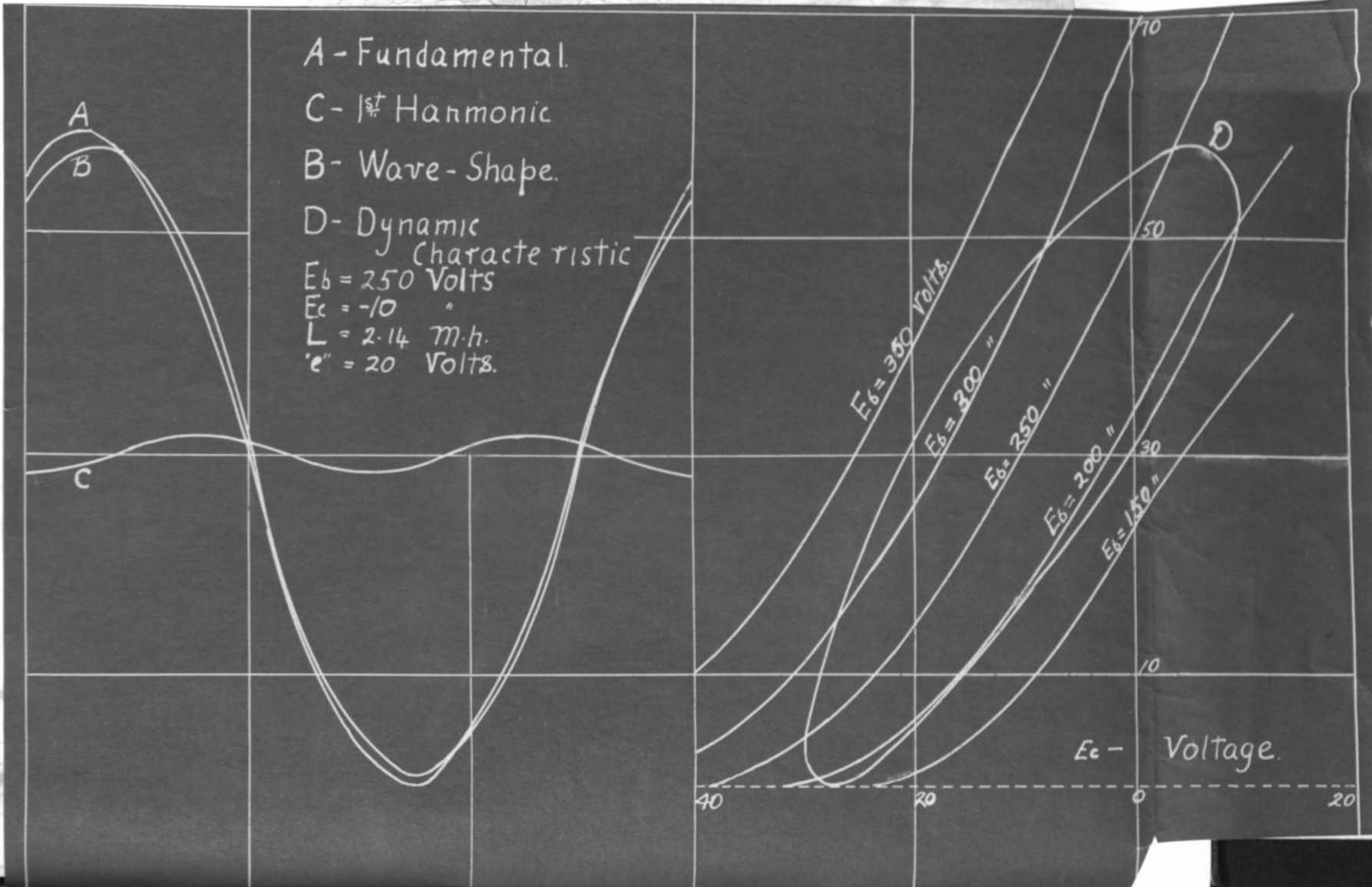
A - Fundamental
B - 1st Harmonic
C - 2nd "
D - 3rd "
E - 4th "
Eb = 250 Volts.
"ε" = 15 " .L = 2.14 M.H.
A
B
C
D
E
8 32
6 24 X 10-1
4 16
10-1 X 2 8
60
40
20
3rd + 4th Harmonic.
1st Harmonic.
Fundamental.
2nd Harmonic
Static Curve
Milamps.
Static.
8
-32
Ec - Volts.
-16
0
16

Fig. 9.



Inductance = 2.14 M.H.
A - Fundamental
B - 1st Harmonic
C 2nd "
D 3rd "
E 4th "
Ec = -10 Volts.
"e" = 20 "
A
B
C
D
E
0
75
175
275
375
Eb - Volts.
4
2

Fig. 10


A - Fundamental.
C - 1st Harmonic
B - Wave - Shape.
D - Dynamic Characteristic
Eb = 250 Volts
Ec = -10 "
L = 2.14 m.h.
"e" = 20 Volts.
A
B
C
D
Eb = 350 Volts.
Eb = 300 "
Eb = 250 "
Eb = 200 "
Eb = 150 "
70
50
30
10
40
20
0
20
Ec - Voltage.